

1. Find the cube of following expressions using formula:

Using the formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

a. Find the cube of $(x + 1)$

Solution:

$$\begin{aligned}\text{Here, } (x + 1)^3 \\&= x^3 + 3x^2(1) + 3x(1)^2 + (1)^3 \\&= x^3 + 3x^2 + 3x + 1 \\&\therefore (x + 1)^3 = x^3 + 3x^2 + 3x + 1\end{aligned}$$

b. Find the cube of $(x - 3)$

Solution:

$$\begin{aligned}\text{Here, } (x - 3)^3 \\&= x^3 + 3x^2(-3) + 3x(-3)^2 + (-3)^3 \\&= x^3 - 9x^2 + 27x - 27 \\&\therefore (x - 3)^3 = x^3 - 9x^2 + 27x - 27\end{aligned}$$

c. Find the cube of $(x + 4)$

Solution:

$$\begin{aligned}\text{Here, } (x + 4)^3 \\&= x^3 + 3x^2(4) + 3x(4)^2 + 4^3 \\&= x^3 + 12x^2 + 48x + 64 \\&\therefore (x + 4)^3 = x^3 + 12x^2 + 48x + 64\end{aligned}$$

d. Find the cube of $(2x - 1)$

Solution:

$$\begin{aligned}\text{Here, } (2x - 1)^3 \\&= (2x)^3 + 3(2x)^2(-1) + 3(2x)(-1)^2 + (-1)^3 \\&= 8x^3 - 12x^2 + 6x - 1 \\&\therefore (2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1\end{aligned}$$

e. Find the cube of $(3x + 2b)$

Solution:

$$\begin{aligned}\text{Here, } (3x + 2b)^3 \\&= (3x)^3 + 3(3x)^2(2b) + 3(3x)(2b)^2 + (2b)^3 \\&= 27x^3 + 54x^2b + 36xb^2 + 8b^3 \\&\therefore (3x + 2b)^3 = 27x^3 + 54x^2b + 36xb^2 + 8b^3\end{aligned}$$

f. Find the cube of $\left(\frac{x}{2} - 1\right)$

Solution:

$$\begin{aligned}\text{Here, } \left(\frac{x}{2} - 1\right)^3 &= \left(\frac{x}{2}\right)^3 + 3\left(\frac{x}{2}\right)^2(-1) + 3\left(\frac{x}{2}\right)(-1)^2 + (-1)^3 \\ &= \frac{x^3}{8} - \frac{3x^2}{4} + \frac{3x}{2} - 1 \\ \therefore \left(\frac{x}{2} - 1\right)^3 &= \frac{x^3}{8} - \frac{3x^2}{4} + \frac{3x}{2} - 1\end{aligned}$$

2. Write the following expressions in the expanded form:

a. Write $(2x + 3y)^3$ in the expanded form.

Solution:

$$\begin{aligned}\text{Here, } (2x + 3y)^3 &= (2x + 3y)(2x + 3y)(2x + 3y) \\ &= (4x^2 + 12xy + 9y^2)(2x + 3y) \\ &= 4x^2(2x + 3y) + 12xy(2x + 3y) + 9y^2(2x + 3y) \\ &= 8x^3 + 12x^2y + 24x^2y + 36xy^2 + 18xy^2 + 27y^3 \\ &= 8x^3 + 36x^2y + 54xy^2 + 27y^3 \\ \therefore (2x + 3y)^3 &= 8x^3 + 36x^2y + 54xy^2 + 27y^3\end{aligned}$$

b. Write $(5a - 8b)^3$ in the expanded form.

Solution:

$$\begin{aligned}\text{Here, } (5a - 8b)^3 &= (5a - 8b)(5a - 8b)(5a - 8b) \\ &= (25a^2 - 80ab + 64b^2)(5a - 8b) \\ &= 25a^2(5a - 8b) - 80ab(5a - 8b) + 64b^2(5a - 8b) \\ &= 125a^3 - 200a^2b - 400a^2b + 640ab^2 + 320ab^2 - 512b^3 \\ &= 125a^3 - 600a^2b + 960ab^2 - 512b^3 \\ \therefore (5a - 8b)^3 &= 125a^3 - 600a^2b + 960ab^2 - 512b^3\end{aligned}$$

3. Write the given expressions in the form of $(a + b)^3$

a. $27a^3 + 108a^2b + 144ab^2 + 64b^3$

Solution:

$$\begin{aligned}\text{Here, } 27a^3 + 108a^2b + 144ab^2 + 64b^3 &= 27a^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3 \\ &= (3a + 4b)^3 \\ \therefore 27a^3 + 108a^2b + 144ab^2 + 64b^3 &= (3a + 4b)^3\end{aligned}$$

b) $8x^3 + 36x^2y + 54xy^2 + 27y^3$

Solution:

$$\begin{aligned} \text{Here, } & 8x^3 + 36x^2y + 54xy^2 + 27y^3 \\ &= (2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3 \\ &= (2x + 3y)^3 \\ \therefore & 8x^3 + 36x^2y + 54xy^2 + 27y^3 = (2x + 3y)^3 \end{aligned}$$

4. Write the expression in the form of $(a-b)^3$.

a. $64m^3 - 48m^2n + 12mn^2 - n^3$

Solution:

$$\begin{aligned} \text{Here, } & 64m^3 - 48m^2n + 12mn^2 - n^3 \\ &= (4m)^3 - 3(4m)^2(n) + 3(4m)(n)^2 - n^3 \\ &= (4m - n)^3 \\ \therefore & 64m^3 - 48m^2n + 12mn^2 - n^3 = (4m - n)^3 \end{aligned}$$

b. $125p^3 - 150p^2q + 60pq^2 - 8q^3$.

Solution:

$$\begin{aligned} \text{Here, } & 125p^3 - 150p^2q + 60pq^2 - 8q^3 \\ &= (5p)^3 - 3(5p)^2(2q) + 3(5p)(2q)^2 - (2q)^3 \\ &= (5p - 2q)^3 \\ \therefore & 125p^3 - 150p^2q + 60pq^2 - 8q^3 = (5p - 2q)^3 \end{aligned}$$

5. Factorize the following:

Use the identities:

$$\checkmark \quad a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\checkmark \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

(a) Factorize: $8x^4 + 27x$

Solution:

$$\begin{aligned} \text{Here, } & \\ &= (2x)^3 + (3)^3x \\ &= x(8x^3 + 27) \\ &= x\{(2x)^3 + 3^3\} \\ &= x(2x + 3)(4x^2 - 6x + 9) \\ \therefore & 8x^4 + 27x = x(2x + 3)(4x^2 - 6x + 9) \end{aligned}$$

(b) Factorize: $x^3 + 8y^3$

Solution:

Here,

$$= x^3 + (2y)^3$$

$$= (x + 2y)(x^2 - 2xy + 4y^2)$$

$$\therefore x^3 + 8y^3 = (x + 2y)(x^2 - 2xy + 4y^2)$$

(c) Factorize: $1 + 125x^3$

Solution:

Here,

$$= 1^3 + (5x)^3$$

$$= (1 + 5x)(1 - 5x + 25x^2)$$

$$\therefore 1 + 125x^3 = (1 + 5x)(1 - 5x + 25x^2)$$

(d) Factorize: $8x^3 + x^3$

Solution:

Here,

$$= (2x)^3 + x^3$$

$$= (2x + x)(4x^2 - 2x^2 + x^2)$$

$$= 3x(3x^2)$$

$$= 9x^3$$

$$\therefore 8x^3 + x^3 = 9x^3$$

(e) Factorize: $(a + b)^3 + 1$

Solution:

Here,

$$= (a + b)^3 + 1^3$$

$$= \{(a + b) + 1\}\{(a + b)^2 - (a + b) + 1\}$$

$$\therefore (a + b)^3 + 1 = (a + b + 1)[(a + b)^2 - (a + b) + 1]$$

(f) Factorize: $x^6 + y^6$

$$= (x^2)^3 + (y^2)^3$$

$$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$$

$$\therefore x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$$

6. Factorize the following expressions:

Using the identity: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(a) Factorize: $250m^4 - 2m$

Solution:

Here,

$$= 2m(125m^3 - 1)$$

$$\begin{aligned}
&= 2m\{(5m)^3 - 1^3\} \\
&= 2m(5m - 1)(25m^2 + 5m + 1) \\
\therefore 250m^4 - 2m &= 2m(5m - 1)(25m^2 + 5m + 1)
\end{aligned}$$

(b) Factorize: $x^3y - 64y^4$

Solution:

Here,

$$\begin{aligned}
&= y(x^3 - 64y^3) \\
&= y\{x^3 - (4y)^3\} \\
&= y(x - 4y)(x^2 + 4xy + 16y^2) \\
\therefore x^3y - 64y^4 &= y(x - 4y)(x^2 + 4xy + 16y^2)
\end{aligned}$$

(c) Factorize: $64p^6q^3 - 125$

Solution:

Here,

$$\begin{aligned}
&= (4p^2q)^3 - 5^3 \\
&= (4p^2q - 5)(16p^4q^2 + 20p^2q + 25) \\
\therefore 64p^6q^3 - 125 &= (4p^2q - 5)(16p^4q^2 + 20p^2q + 25)
\end{aligned}$$

(d) Factorize: $(a - b)^3 - 8(a + b)^3$

Solution:

Here,

$$\begin{aligned}
&= (a - b)^3 - \{2(a + b)\}^3 \\
&= \{(a - b) - 2(a + b)\}\{(a - b)^2 + 2(a - b)(a + b) + 4(a + b)^2\} \\
&= (-a - 3b)\{(a - b)^2 + 2(a - b)(a + b) + 4(a + b)^2\} \\
\therefore (a - b)^3 - 8(a + b)^3 &= (-a - 3b)\{(a - b)^2 + 2(a - b)(a + b) + 4(a + b)^2\}
\end{aligned}$$

(e) Factorize: $x^3y^3 - y^3x^3$

Solution:

Here,

$$\begin{aligned}
&= (xy)^3 - (xy)^3 \\
&= 0 \\
\therefore x^3y^3 - y^3x^3 &= 0
\end{aligned}$$

(f) Factorize: $p^3 - \frac{1}{p^3}$

Solution:

Here,

$$\begin{aligned}
&= \left(p - \frac{1}{p}\right) \left\{p^2 + p\left(\frac{1}{p}\right) + \frac{1}{p^2}\right\} \\
&= \left(p - \frac{1}{p}\right) \left(p^2 + 1 + \frac{1}{p^2}\right) \\
\therefore p^3 - \frac{1}{p^3} &= \left(p - \frac{1}{p}\right) \left(p^2 + 1 + \frac{1}{p^2}\right)
\end{aligned}$$

7. How many cubic feet wood is remain if we cut a piece of cubical wood having length 7 ft from a cubical log of x feet length? ($x > 7$ ft)

Solution:

Here,

Length of the cubical log = x ft

Length of the cubical piece cut = 7 ft

Required: Volume of wood remaining.

Now,

Volume of the original cubical log = $x^3 \text{ ft}^3$

Volume of the cubical piece cut = $7^3 \text{ ft}^3 = 343 \text{ ft}^3$

Volume of wood remaining

$$= x^3 - 343$$

$$= x^3 - 7^3$$

$$= (x - 7)(x^2 + 7x + 49)$$

$\therefore$ The remaining wood = $(x - 7)(x^2 + 7x + 49)$ cubic feet.