Irrational Numbers

Introduction

An irrational number is a number that cannot be expressed in the form $\left(\frac{p}{q}\right)$, where (p) and (q) are integers and $(q \neq 0)$.

The decimal expansion of an irrational number is non-terminating and non-repeating.

Definition

A number is called an irrational number if:

- It cannot be written as $\left(\frac{p}{q}\right)$, where (p) and (q) are integers and $(q \neq 0)$.
- Its decimal form neither terminates nor repeats.

Examples of Irrational Numbers

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,

Some decimal forms:

- $(\sqrt{2} = 1.414213562 \ldots)$
- $(\sqrt{3} = 1.732050807 \ldots)$
- $(\pi = 3.141592653 \ldots)$

These decimals continue forever without any repeating pattern.

Why are They Called Irrational?

Consider:

$\sqrt{2} = 1.414213562$

Since $(\sqrt{2})$cannot be written as a fraction of two integers, it is an irrational number.

Properties of Irrational Numbers

1. Their decimal expansion is non-terminating and non-repeating.
2. They cannot be expressed as $\left(\frac{p}{q}\right)$.
3. The square root of a non-perfect square is irrational.
4. The sum of a rational and an irrational number is irrational.
5. The product of a non-zero rational number and an irrational number is irrational.

Examples and non-examples

Irrational Numbers	Rational Numbers
$(\sqrt{2})$	$\left(\frac{1}{2}\right)$
$(\sqrt{3})$	(0.25)
$(\sqrt{7})$	(5)
(π)	(-3)
$(\sqrt{11})$	$(0.333 \ldots)$

Important Note

- Square roots of perfect squares are rational.
  - $(\sqrt{4} = 2)$
  - $(\sqrt{9} = 3)$
  - $(\sqrt{25} = 5)$
- Square roots of non-perfect squares are irrational.
  - $(\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7})$

1. Solved Examples

Example 1

State whether $(\sqrt{49})$is rational or irrational.

Solution:

$\sqrt{49} = 7$

Since 7 is an integer, $(\sqrt{49})$ is a rational number.

Example 2

State whether $(\sqrt{10})$ is rational or irrational.

Solution:

10 is not a perfect square.

$\sqrt{10} = 3.162277$

The decimal is non-terminating and non-repeating.

Therefore, $(\sqrt{10})$ is an irrational number.

B. Representation of Irrational Numbers on the Number Line

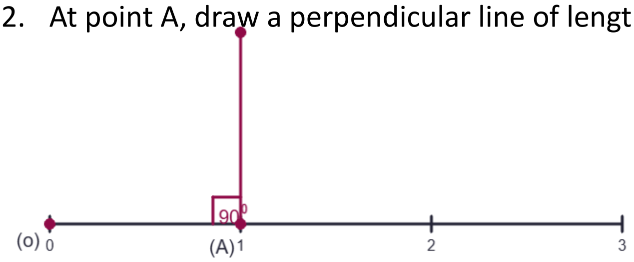
An irrational number can be represented on the number line using geometric construction.

Example: Representing $(\sqrt{2})$ on the Number Line

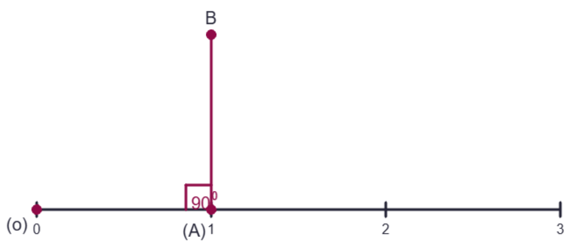
Steps

1. Draw a number line and mark point O (0) and A (1).

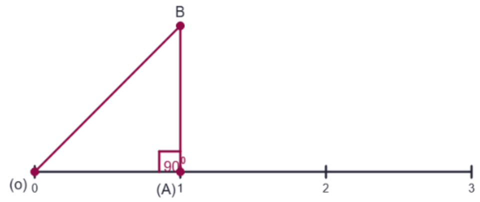
2. At point A, draw a perpendicular line of length 1 unit.



3. Let the end of the perpendicular be B.



4. Join O and B.



The triangle OAB is a right-angled triangle with:

$OA = 1, \quad AB = 1$

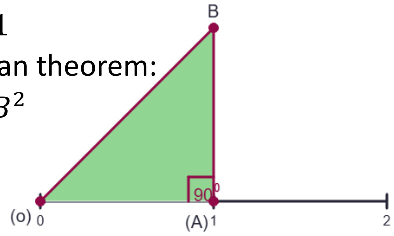
Using the Pythagorean theorem:

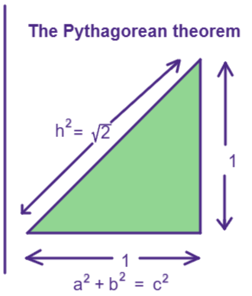
or, $OA^2 + AB^2 = OB^2$

or, $1^2 + 1^2 = OB^2$

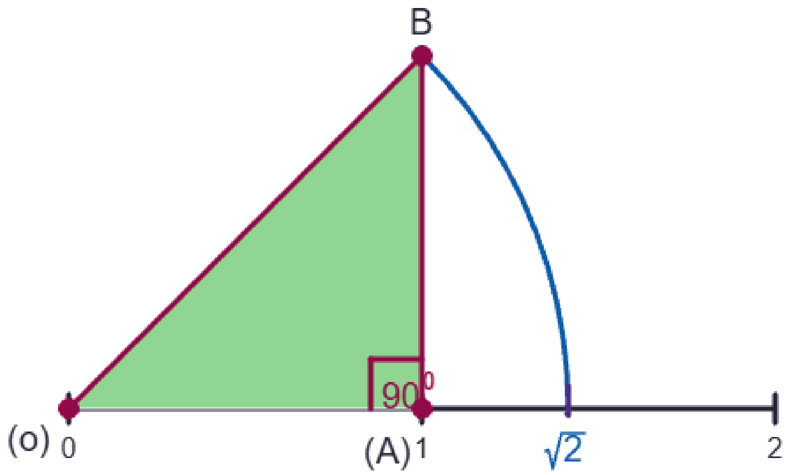
or, $2 = OB^2$

$\therefore OB = \sqrt{2}$





5. With O as the center and OB as the radius, draw an arc cutting the number line at point P. Then $OP = (\sqrt{2})$.



Similarly

- $(\sqrt{3}), (\sqrt{5}), (\sqrt{6})$, etc., can also be represented using successive right triangles.
- The value of (π)can be approximately represented as 3.14 on the number line.

Important Fact

Every irrational number corresponds to a unique point on the number line, just like rational numbers. Therefore, irrational numbers can be represented geometrically on the number line.

GeoGebra file:

Sqare root Spiral.ggb